The Classical Electrodynamics Approach
To
Explain The Photoelectric Effect And The Photoelectric Emission

By re-analyzing the photoelectric effect, the limitations of Einstein's photon hypothesis are identified and the relation between the photoelectron's kinetic energy and the circular frequency of the incident light is reinterpreted using classical electrodynamics. And it is realized that photoelectric emission is dependent on the incident light's circular frequency ω and the photon density of the incident light.

Key words: photoelectric effect, photon hypothesis, classical electrodynamics, photoelectric emission, photon density.

The limitation of the Photon Hypothesis

According to the electromagnetic theory of light, its energy is related to the amplitude of the electric field of the electromagnetic wave, $w = \varepsilon E^2$ (where E is the amplitude). It apparently has nothing to do with the light's circular frequency ω.
To explain the photoelectric effect, Einstein put forward the photon hypothesis. His paper hypothesized light was made of quantum packets of energy called photons. Each photon carried a specific energy related to its circular frequency ω, $E = \hbar\omega$. This has nothing to do with the amplitude of the electromagnetic wave.

First, for the electromagnetic wave that the amplitude E has nothing to do with the light's circular frequency ω, if the light's circular frequency ω is high enough, the energy of the photon in light is greater than the light's energy, $\hbar\omega \gg \varepsilon E^2$. Apparently, this is incompatible with the electromagnetic theory of light.

Secondly, if light has corpuscular properties, according to the law of conservation of momentum, the direction of motion of the electron will be the same as that of the photon (light) after the collision. But in the photoelectric effect, the direction of motion of the liberated photoelectron is contrary to this direction. (Figure 1)
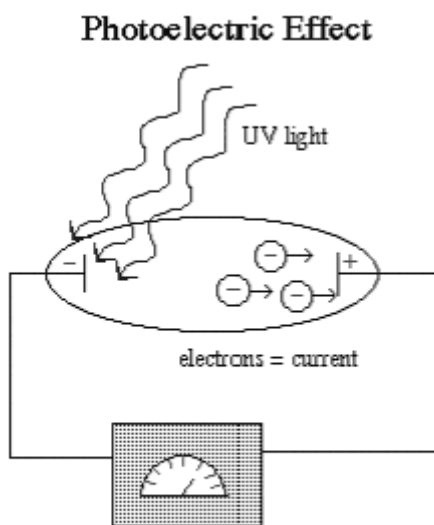

Figure 1

Light moves toward the negative pole, but the liberated photoelectrons move toward the positive pole. Einstein's photon hypothesis cannot resolve this problem.

Thirdly, Einstein's photon hypothesis cannot explain the two and the muitl- photon photoelectric emission.
In 1968, M.C.Teich and G.J.Wolga reported the observation of two-photon photoelectric emission from sodium metal[1].
In their paper, they indicated that the electrons might be emitted even though the incident photons with energies less than the threshold energy of the sodium metal.
This is the two-photon photoelectric emission.

Reinterpretation

To eject an electron from a metal plate requires a minimal threshold energy of W,
The author shows that the minimal threshold energy W could be transferred by the power flux density (Poynting Vector S) of the near zone electromagnetic field of the electric dipole.

$$S = \omega \frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2 \theta)}{32\pi^2 \varepsilon r^5} \sin(2\omega t + \pi) \tag{a}$$

S increases with the increase of the ω. When $S > W$, electrons are emitted. Below a certain frequency, no electrons will be emitted regardless of the intensity of the incident light.

When a plane electromagnetic wave is directed at the negative pole, the electron in the metal will oscillate with the electric vector of the incident plane electromagnetic wave
[$E(x,t) = E_0 \cos(kx - \omega t)$, ω = circular frequency]. Its oscillating frequency is equal to the incident plane electromagnetic wave's circular frequency ω. The electron will radiate an electromagnetic wave when it performs a simple harmonic oscillation, this electromagnetic wave's circular frequency is equal to the oscillating frequency ω of the electron.
The electron will be exposed to an Ampere force by the magnetic vector of the incident plane electromagnetic wave [$B(x,t) = B_0 \cos(kx - \omega t)$]. The direction of the Ampere force is the same as the direction of motion of the incident plane electromagnetic wave.
The electron will not leave the metal due to the electric and magnetic vector of the incident plane electromagnetic wave.

The electron's simple harmonic oscillation can be considered as an electric dipole. To simplify the calculation, the system of Figure 2 is used. It consists of two conductor balls that are connected by a thin lead of length, Δl.
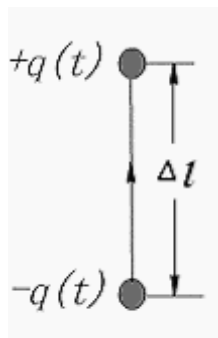

Firgue 2

Suppose the charge $q(t)$ changes as the sine with the time,

$$q(t) = R_e[Q e^{j\omega t}] \tag{1}$$

[1] M. C. Teich and G. J. Wolga, "Two-Quantum Volume Photoelectric Effect in Sodium," Phys. Rev. 171, 809-814 (1968)

the current is

$$i(t) = \frac{dq(t)}{dt} = I \cos \omega t = R_e \left[I e^{j\omega t} \right] \tag{2}$$
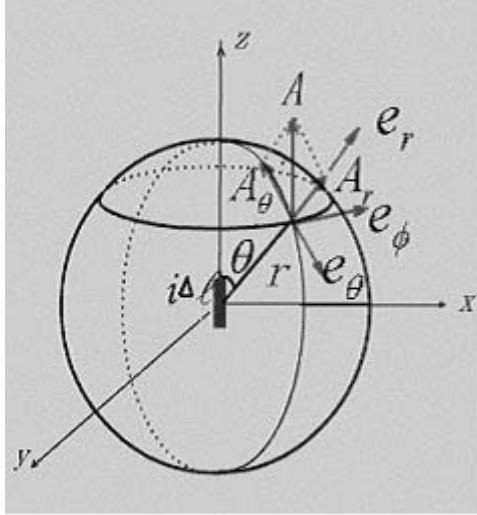
$$I = j\omega Q \tag{3}$$

Figure 3 follows,

Figure 3

The electric dipole lies on the Z axis, and is locate on the origin of the coordinate, the length of the thin lead is Δl, the area of the cross section of the thin lead is ΔS, therefore the cube is $dV = l\Delta S$,

So

$$JdV = e_z \frac{I}{\Delta S} \cdot \Delta l \Delta S = e_z I \Delta l \tag{4}$$

$$J(t) = J \cos \omega t \tag{5}$$

and the retarded potential $A(r,t)$ is

$$A(r,t) = \frac{u}{4\pi} \int \frac{Je^{-jkr}}{r} dV' \cos \omega t = \frac{u}{4\pi} \int \frac{e_z Ie^{-jkr}}{r} dz' \cos \omega t \tag{6}$$

Because $r \gg \Delta l$, $A(r,t)$ can be expressed as the following:

$$A(r,t) = e_z \frac{u}{4\pi r} I\Delta le^{-jkr} \cos \omega t \tag{7}$$

Then the three spherical coordinate components of the retarded potential $A(r,t)$ are

$$A_r(t) = A_z \cos \theta = \frac{uI\Delta l}{4\pi r} \cos \theta e^{-jkr} \cos \omega t \tag{8}$$

3

$$A_\theta(t) = -A_z \sin\theta = -\frac{uI\Delta l}{4\pi r}\sin\theta e^{-jkr}\cos\omega t \tag{9}$$

$$A_\phi(t) = 0 \tag{10}$$

Because

$$B = \nabla \times A(r,t) \tag{11}$$

$$E = \frac{1}{j\omega\varepsilon}\nabla \times H \tag{12}$$

We can obtain

$$H_\phi(t) = \frac{k^2 I\Delta l \sin\theta}{4\pi}\left[\frac{j}{kr} + \frac{1}{(kr)^2}\right]e^{-jkr}\cos\omega t \tag{13}$$

$$E_r(t) = \frac{2I\Delta l k^3 \cos\theta}{4\pi\omega\varepsilon}\left[\frac{1}{(kr)^2} - \frac{j}{(kr)^3}\right]e^{-jkr}\cos\omega t \tag{14}$$

$$E_\theta(t) = \frac{2I\Delta l k^3 \sin\theta}{4\pi\omega\varepsilon}\left[\frac{j}{kr} + \frac{1}{(kr)^2} - \frac{j}{(kr)^3}\right]e^{-jkr}\cos\omega t \tag{15}$$

$$E_\phi(t) = H_r(t) = H_\theta(t) = 0 \tag{16}$$

In the near zone ($r \ll \lambda$), $kr \ll 1$, so

$$\frac{1}{kr} \ll \frac{1}{(kr)^2} \ll \frac{1}{(kr)^3}, \quad e^{-jkr} \approx 1$$

We can obtain

$$E_r(t) = -j\frac{I\Delta l \cos\theta}{\omega 2\pi\varepsilon r^3}\cos\omega t \tag{17}$$

$$E_\theta(t) = -j\frac{I\Delta l \sin\theta}{\omega 4\pi\varepsilon r^3}\cos\omega t \tag{18}$$

$$H_\phi(t) = \frac{I\Delta l \sin\theta}{4\pi r^2}\cos\omega t \tag{19}$$

Because

$$i(t) = \frac{dq(t)}{dt} = I\cos\omega t = R_e\left[Ie^{j\omega t}\right] \tag{2}$$

$$I = j\omega Q \tag{3}$$

We can obtain

$$I = \frac{\omega Q \cos(\omega t + \frac{\pi}{2})}{\sin(\omega t + \frac{\pi}{2})} \tag{20}$$

$$E_r(t) = -j\frac{I\Delta l \cos\theta}{\omega 2\pi\varepsilon r^3}\cos\omega t = \frac{Q\Delta l}{2\pi\varepsilon r^3}\cos\theta\cos\omega t \tag{21}$$

$$E_\theta(t) = -j\frac{I\Delta l \sin\theta}{\omega 4\pi\varepsilon r^3}\cos\omega t = \frac{Q\Delta l}{4\pi\varepsilon r^3}\sin\theta\cos\omega t \tag{22}$$

$$H_\phi(t) = \frac{\omega Q\Delta l \sin\theta}{4\pi r^2}\cos(\omega t + \frac{\pi}{2}) \tag{23}$$

So the near zone electromagnetic field of the electric dipole is:

$$E_r(t) = \frac{Q\Delta l \cos\theta}{2\pi\varepsilon r^3}\cos\omega t \tag{24}$$

$$E_\theta(t) = \frac{Q\Delta l \sin\theta}{4\pi\varepsilon r^3}\cos\omega t \tag{25}$$

$$H_\phi(t) = \frac{\omega Q\Delta l \sin\theta}{4\pi r^2}\cos(\omega t + \frac{\pi}{2}) \tag{26}$$

$$E_\phi(t) = H_r(t) = H_\theta(t) = 0 \tag{16}$$

Therefore, the components of the Poynting Vector $S = E \times H$ are

$$S_r = E_\theta(t) \cdot H_\phi(t) = \frac{\omega Q^2 \Delta l^2 \sin^2\theta}{32\pi^2\varepsilon r^5}\sin(2\omega t + \pi) \tag{27}$$

$$S_\theta = E_r(t) \cdot H_\phi(t) = \frac{\omega Q^2 \Delta l^2 \sin\theta\cos\theta}{16\pi^2\varepsilon r^5}\sin(2\omega t + \pi) \tag{28}$$

and

$$S = S_r + S_\theta = \omega\frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2\theta)}{32\pi^2\varepsilon r^5}\sin(2\omega t + \pi) \tag{29}$$

Poynting Vector S is an instantaneous value, therefore the electron that in the near zone electromagnetic field of the electric dipole can obtain the energy

$$S = \omega\frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2\theta)}{32\pi^2\varepsilon r^5}\sin(2\omega t + \pi) \tag{a}$$

S increases with the increase of ω.

The minimal threshold energy W and the kinetic energy of the photoelectron could be transferred by the Poynting Vector S. According to the law of conservation of energy, we obtain

$$\omega\frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2\theta)}{32\pi^2\varepsilon r^5}\sin(2\omega t + \pi) = \frac{1}{2}mv^2 + W \tag{b}$$

Below a certain frequency, no electrons will be emitted, no matter how high the intensity of the incident light. This formula has explained the proportional relationship between the photoelectron's kinetic energy and the circular frequency of the incident light.

Discussion

In fact, in the photoelectric effect, the intensity of the incident light is only related to the number of photons. It has nothing to do with the incident light's electric vector amplitude. The incident light's electric vector amplitude will increase the photoelectron's energy. It will not increase the number of photoelectrons that is in direct ratio to the intensity of the incident light. The increase of the number of photons will not increase the amplitude (Δl) of the electron.

Thus the amplitude (Δl) of the electron which acts as a simple harmonic oscillation does not increase, and the Poynting Vector S does not increase with the increase of the intensity of incident light.

The velocity V of the electron which acts as a simple harmonic oscillation in the metal is $V \ll C$, where C is the velocity of light in the vacuum, and

$$V = \frac{dx}{dt} = \omega \Delta l \cos(\omega t + \frac{\pi}{2})$$ **(30)**

so

$$\omega \Delta l \ll C, \ \Delta l \ll \frac{C}{\omega}.$$ **(31)**

$$\Delta l \ll \frac{\lambda}{2\pi}$$ **(32)**

Therefore, the electron can be considered an electric dipole.

The electron which in the near zone ($r \ll \lambda$) electromagnetic field of the electric dipole can obtain the energy

$$S = \omega \frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2 \theta)}{32\pi^2 \varepsilon r^5} \sin(2\omega t + \pi)$$ **(a)**

S increases with the increase of ω. Below a certain frequency, no electrons will be emitted, no matter how high the intensity of the incident light.

If the electron is in the near zone electromagnetic field of many electric dipoles, the electron will obtain the resultant Poynting Vector $\sum_n S$

$$\sum_n S =$$
$$\omega \frac{Q^2 \Delta l^2 \sin(2\omega t + \pi)}{32\pi^2 \varepsilon} [\frac{(\sin 2\theta_1 + \sin^2 \theta_1)}{r_1^5} + \frac{(\sin 2\theta_2 + \sin^2 \theta_2)}{r_2^5} + ... + \frac{(\sin 2\theta_n + \sin^2 \theta_n)}{r_n^5}]$$ **(c)**

Where n is the number of the electric dipoles.

$\sum_n S$ increases with the increase of ω and the number of the electric dipoles.

When $\sum_n S > W$, electrons are emitted. Below the minimal threshold energy of W, no electrons will be emitted.

The minimal threshold energy W and the kinetic energy of the photoelectron could be transferred by the resultant Poynting Vector $\sum_n S$. According to the law of conservation of energy, we obtain

$$\sum_{n} S = \frac{1}{2}mv^2 + W \tag{d}$$

$$\omega \frac{Q^2 \Delta l^2 \sin(2\omega t + \pi)}{32\pi^2 \varepsilon} \left[\frac{(\sin 2\theta_1 + \sin^2 \theta_1)}{r_1^5} + \frac{(\sin 2\theta_2 + \sin^2 \theta_2)}{r_2^5} + \ldots + \frac{(\sin 2\theta_n + \sin^2 \theta_n)}{r_n^5}\right] \tag{e}$$

$$= \frac{1}{2}mv^2 + W$$

Below a certain frequency or a certain number of the electric dipoles, no electrons will be emitted.

We know, the laser is brighter than light in general, therefore, we can consider that the photon density (the number of the photon per unit area) of the laser is greater than that of general light.
Suppose the photon density of the general light is one per unit area, and then the photon density of the laser is two or more per unit area.

When a beam of light is directed at the metal surface, the electron in the metal surface will absorb a photon and perform a simple harmonic oscillation. The electron's oscillation can be considered an electric dipole that will radiate an electromagnetic wave.

The photon density of the general light is only one per unit area, so the electrons in the metal surface per unit area will only absorb one photon, even if there are many electrons in that surface.
The electron in the near zone electromagnetic field of the electric dipole can obtain the energy,

$$S = \omega \frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2 \theta)}{32\pi^2 \varepsilon r^5} \sin(2\omega t + \pi) \tag{a}$$

S increases with the increase of ω only, it is the proportional relationship between the

photocurrent J and the intensity of the incident general light I_0, namely: $J \propto I_0$ **(A1)**

This is the photoelectric effect.

When a beam of laser light is directed at the metal surface, the photon density of the laser is two or more per unit area, so two electrons in the metal surface per unit area will absorb two or more photons (every electron absorbs a photon), these electrons can be considered electric dipoles.
These two electrons can be considered two electric dipoles.

The electron in the near zone electromagnetic field of these two electric dipoles can obtain the energy

$$\sum_{2} S =$$

$$\omega \frac{Q^2 \Delta l^2 \sin(2\omega t + \pi)}{32\pi^2 \varepsilon} \left[\frac{(\sin 2\theta_1 + \sin^2 \theta_1)}{r_1^5} + \frac{(\sin 2\theta_2 + \sin^2 \theta_2)}{r_2^5}\right] \tag{f}$$

$\sum_{2} S$ increases with the increase of ω and the photon density of the laser.

The electron can obtain two electric dipoles' resultant near zone electromagnetic energy, even if the incident laser's circular frequency is less than the metal's threshold circular frequency, This is due to the proportional relationship between the photocurrent J and the

two times of the intensity of the incident laser I_0, namely: $J \propto 2I_0$.   **(A2)**

This is the two-photon photoelectric emission.

When the photon density of the laser is n, n electrons in the metal surface per unit area will absorb n photons, n electrons can be considered as electric dipoles.

The electron that in the near zone electromagnetic field of these electric dipoles can obtain the energy

$$\sum_n S =$$

$$\omega \frac{Q^2 \Delta l^2 \sin(2\omega t + \pi)}{32\pi^2 \varepsilon} [\frac{(\sin 2\theta_1 + \sin^2 \theta_1)}{r_1^5} + \frac{(\sin 2\theta_2 + \sin^2 \theta_2)}{r_2^5} + ... + \frac{(\sin 2\theta_n + \sin^2 \theta_n)}{r_n^5}] \quad \text{(c)}$$

$\sum_n S$ increases with the increase of ω and the photon density of the laser.

The electron can obtain n electric dipoles' near zone electromagnetic energy, so even if the incident laser's circular frequency is less than that of the metal, electrons will be emitted. The proportional relationship between the photocurrent J and the n times of the intensity of incident laser I_0 is: $J \propto nI_0$, **(A2)**
This is the muitl- photon photoelectric emission.

According to the formula
$$S = \omega \frac{Q^2 \Delta l^2 (\sin 2\theta + \sin^2 \theta)}{32\pi^2 \varepsilon r^5} \sin(2\omega t + \pi) \quad \text{(a)}$$
Remarkably, r^5 increases with the increase of r, so S decrease with the increase of r.

Therefore, the electron can only obtain enough near zone electromagnetic energy from the nearer electric dipoles and insufficient near zone electromagnetic energy from the further electric dipoles.

In their paper, M.C.Teich and G.J.Wolga also reported that the proportional relationship between the photocurrent density J and the second power of the intensity of the incident laser I_0, namely: $J \propto I_0^2$, I_0 expressed in W/cm^2, and I_0^2 expressed in W^2/cm^4, and therefore $i^{(2)} \propto I_0^2 A$. Since $I_0 = P/A$, however, $i^{(2)} \propto P^2/A$. **(33)**

But even if an electron can absorbs two or many photons, it needs two or more photons to emit an electron, thus it needs to increase the intensity of the incident laser I_0, the intensity of the incident light I_0 is related to the number of photons, the increase of the number of photons will not change the unit of the intensity of the incident laser I_0, I_0 still expressed in W/cm^2, not expressed in W^2/cm^4, the increase of the number of photons will increase the photocurrent density J, it means that that the proportional relationship between the photocurrent density J and the intensity of the incident laser I_0 is still $J \propto I_0$, not $J \propto I_0^2$. Therefore, the author thinks that their result is inconsequential.

In addition, in their paper, the two-quantum photocurrent is
$$i^{(2)} = \pi e \rho \{\frac{2N-1}{N}\}[\frac{dr_0^2 mc^2 (\beta P)^2}{(2\pi)^2 2(h\nu)^3 \nu A}] \times [\frac{4\pi}{3}(\frac{E_F}{2h\nu})k_F(1 + \frac{e\varphi}{E_F} - \frac{2h\nu}{E_F})^{3/2}] \quad \text{(34)}$$

$$i^{(2)} \propto \frac{P^2}{(h\nu)^3 A} \quad \text{(35)}$$
The symbols have the following interpretation:
$i^{(2)}$ is the two-quantum photocurrent, r_0 is the classical electron radius, P is the incident laser power, A is the area of the illuminated spot, $e\varphi$ is the work function of the material, E_F is the Fermi energy of the material, k_F is the wave number of an electron at the Fermi

surface, N is the number of mode in which the laser is oscillating, ν is the frequency of the incident laser, d is the escape depth or photoelectron range, β is 1 minus the reflectivity, and ρ is the two-quantum "oscillator strength".

In fact, according to the quantum theory,
$$I_0 \propto h\nu , P \propto h\nu , \tag{36}$$

Therefore, $i^{(2)} \propto \dfrac{P^2}{(h\nu)^3 A} \propto \dfrac{1}{I_0}$, $J \propto \dfrac{1}{I_0}$. $\tag{37}$

The formula

$$i^{(2)} = \pi e \rho \{ \frac{2N-1}{N} \} [\frac{dr_0^{\,2} mc^2 (\beta P)^2}{(2\pi)^2 2(h\nu)^3 \nu A}] \times [\frac{4\pi}{3} (\frac{E_F}{2h\nu}) k_F (1 + \frac{e\varphi}{E_F} - \frac{2h\nu}{E_F})^{3\!/\!2}] \tag{34}$$

is unreasonable, it cannot explain the truth that the photocurrent density J increases with the increase of the intensity of incident light I_0.

Conclusion

Einstein's photon hypothesis is incompatible with the electromagnetic theory of light, and it cannot resolve the problem of the direction of motion of the liberated electron, and cannot explain the two and the muitl- photon photoelectric emission.

From the above analysis, it is also realized that the photoelectric emission is dependent on the incident light's circular frequency ω and the photon density (the number of the photon per unit area) of the incident light.

Finally, to resolve the illogicality between Einstein's photon hypothesis and the electromagnetic theory of light, the author proposes the photon vibrator concept instead of Einstein's photon. We know the wave is the vibrator's propagation course so it is reasonable to consider that the light consists of photon vibrators, but this photon vibrator is not Einstein's photon. Its energy has nothing to do with frequency but relates only to its amplitude.

Reference:
Classical Electrodynamics by J.D. Jackson   Chapter 9.

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